SM3 12.2: Basic Trig Proof

Use two columns to prove each identity.

1)

$4(3-5)^2 = 16$	Given
$4(-2)^2 = 16$	Subtraction
$4(4) = 16$	Multiplication
$16 = 16$	Multiplication
QED	

2)

$(6-9)(2-8) = 18$	Given
$(-3)(-6) = 18$	Subtraction
$18 = 18$	Multiplication
QED	

3)

$\sec(\theta)\cos(\theta) = 1$	Given
$\frac{1}{\cos(\theta)}\cos(\theta) = 1$	Def of sec
$1 = 1$	Division
QED	

4)

$\frac{1}{\sin(\theta)} = \csc(\theta)$	Given
$\csc(\theta) = \csc(\theta)$	Def of csc
QED	

5)

$3\tan(\theta) + 4\tan(\theta) = 7\tan(\theta)$	Given
$7\tan(\theta) = 7\tan(\theta)$	Addition
QED	

6)

$-\tan(\theta)\csc(\theta) = -\sec(\theta)$	Given
$-\frac{\sin(\theta)}{\cos(\theta)}\frac{1}{\sin(\theta)} = -\sec(\theta)$	Def of tan, csc
$-\frac{1}{\cos(\theta)} = -\sec(\theta)$	Divide
$-\sec(\theta) = -\sec(\theta)$	Def of sec
QED	

7)

$\sin(\theta)\cot(\theta) = \cos(\theta)$	Given
$\sin(\theta)\frac{\cos(\theta)}{\sin(\theta)} = \cos(\theta)$	Def of cot
$\cos(\theta) = \cos(\theta)$	Divide
QED	

8)

$\cos(\theta)\tan(\theta) = \sin(\theta)$	Given
$\cos(\theta)\frac{\sin(\theta)}{\cos(\theta)} = \sin(\theta)$	Def of tan
$\sin(\theta) = \sin(\theta)$	Divide
QED	

9) $\sin(\theta)\cot(\theta)\tan(\theta) = \sin(\theta)$ Given

$\sin(\theta)\frac{\cos(\theta)}{\sin(\theta)}\frac{\sin(\theta)}{\cos(\theta)} = \sin(\theta)$ Def of cot, tan

$\sin(\theta) = \sin(\theta)$ Divide

QED

10) $\tan^2(\theta)\cos^4(\theta) = \sin^2(\theta)\cos^2(\theta)$ Given

$\frac{\sin^2(\theta)}{\cos^2(\theta)}\cos^4(\theta) = \sin^2(\theta)\cos^2(\theta)$ Def of tan

$\sin^2(\theta)\cos^2(\theta) = \sin^2(\theta)\cos^2(\theta)$ Divide

QED

11) $(\sin(\theta)+1)(\sin(\theta)-1) = \sin^2(\theta)-1$ Given

$\sin^2\theta - \sin\theta + \sin\theta - 1 = \sin^2\theta - 1$ Distribute

$\sin^2(\theta) - 1 = \sin^2(\theta) - 1$ Addition

QED

12) $(\cos(\theta)+1)(\cos(\theta)+4) = \cos^2(\theta) + 5\cos(\theta) + 4$ Given

$\cos^2(\theta) + 4\cos(\theta) + \cos(\theta) + 4 = \cos^2(\theta) + 5\cos(\theta) + 4$ Distribute

$\cos^2(\theta) + 5\cos(\theta) + 4 = \cos^2(\theta) + 5\cos(\theta) + 4$ Add

QED

13) $\tan^2(\theta) - 9 = (\tan(\theta)-3)(\tan(\theta)+3)$ Given

$(\tan(\theta)-3)(\tan(\theta)+3) = (\tan(\theta)-3)(\tan(\theta)+3)$ Factor

QED

14) $(\csc(\theta)-2)(\csc(\theta)+2)+4 = \frac{1}{\sin^2(\theta)}$ Given

$\csc^2(\theta) + 2\csc\theta - 2\csc\theta - 4 + 4 = \frac{1}{\sin^2(\theta)}$ Distribute

$\csc^2(\theta) = \frac{1}{\sin^2(\theta)}$ Add

$\frac{1}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)}$ Def of csc

QED

15) $\frac{\sin^2(\theta)+10\sin(\theta)+24}{\sin^2(\theta)-16}=\frac{\sin(\theta)+6}{\sin(\theta)-4}$ Given

$\frac{(\sin(\theta)+4)(\sin(\theta)+6)}{(\sin(\theta)+4)(\sin(\theta)-4)}=\frac{\sin(\theta)+6}{\sin(\theta)-4}$ Factor

$\frac{\sin(\theta)+6}{\sin(\theta)-4}=\frac{\sin(\theta)+6}{\sin(\theta)-4}$ Divide

QED

16) $\frac{2\cot^2(\theta)-5\cot(\theta)-3}{\cot^2(\theta)-10\cot(\theta)+21}=\frac{2\cot(\theta)+1}{\cot(\theta)-7}$ Given

$\frac{(2\cot(\theta)+1)(\cot(\theta)-3)}{(\cot(\theta)-3)(\cot(\theta)-7)}=\frac{2\cot(\theta)+1}{\cot(\theta)-7}$ Factor

$\frac{2\cot(\theta)+1}{\cot(\theta)-7}=\frac{2\cot(\theta)+1}{\cot(\theta)-7}$ Divide

QED

17) $\frac{\cos^2(\theta)+\sin^2(\theta)}{\sin^2(\theta)}=\cot^2(\theta)+1$ Given

$\frac{\cos^2(\theta)}{\sin^2(\theta)}+\frac{\sin^2(\theta)}{\sin^2(\theta)}=\cot^2(\theta)+1$ Divide

$\cot^2(\theta)+1=\cot^2(\theta)+1$ Def of cot/divide

QED

18) $\frac{\csc(\theta)+3\sec(\theta)}{5\csc(\theta)}=\frac{1}{5}+\frac{3}{5}\tan(\theta)$ Given

$\frac{\csc(\theta)}{5\csc(\theta)}+\frac{3\sec(\theta)}{5\csc(\theta)}=\frac{1}{5}+\frac{3}{5}\tan(\theta)$ Divide

$\frac{1}{5}+\frac{3\sin(\theta)}{5\cos(\theta)}=\frac{1}{5}+\frac{3}{5}\tan(\theta)$ Divide, def of sec,csc

$\frac{1}{5}+\frac{3}{5}\tan(\theta)=\frac{1}{5}+\frac{3}{5}\tan(\theta)$ Def of tan

QED

19)

$$\tan(\theta) + \sec(\theta) = \frac{\sin(\theta) + 1}{\cos(\theta)} \quad \text{Given}$$

$$\frac{\sin(\theta)}{\cos(\theta)} + \frac{1}{\cos(\theta)} = \frac{\sin(\theta) + 1}{\cos(\theta)} \quad \text{Def of tan, sec}$$

$$\frac{\sin(\theta) + 1}{\cos(\theta)} = \frac{\sin(\theta) + 1}{\cos(\theta)} \quad \text{Add}$$

QED

20)

$$\sec(\theta) + \sin(\theta) = \frac{1 + \sin(\theta)\cos(\theta)}{\cos(\theta)} \quad \text{Given}$$

$$\frac{1}{\cos(\theta)} + \sin(\theta) = \frac{1 + \sin(\theta)\cos(\theta)}{\cos(\theta)} \quad \text{Def of sec}$$

$$\frac{1}{\cos(\theta)} + \frac{\sin(\theta)\cos(\theta)}{\cos(\theta)} = \frac{1 + \sin(\theta)\cos(\theta)}{\cos(\theta)} \quad \text{Mult Id.}$$

$$\frac{1 + \sin(\theta)\cos(\theta)}{\cos(\theta)} = \frac{1 + \sin(\theta)\cos(\theta)}{\cos(\theta)} \quad \text{Add}$$

QED